

2nd JOINT MATHEMATICAL
MEETING OF
SERBIA AND MONTENEGRO

Belgrade, 26-28 January 2023

Welcome Address

The idea of organizing the Joint mathematical meeting of Serbia and Montenegro came from a group of mathematicians of the younger generation from Podgorica and Belgrade. After the breakup of the state union of Serbia and Montenegro, the mathematical collaboration between the two successor states has diminished to the point of near-vanishing, reduced only to a few individual contacts. The first Joint mathematical meeting of Serbia and Montenegro was successfully organized in Budva in 2019. Since that, the exchange of researchers between countries has increased, mutual supervision of PhD and master students has started, and the first joint projects have been implemented.

However, the outbreak of the COVID-19 pandemic prevented us from organizing the second one. The pandemic negatively impacted scientific collaboration and the relationships between mathematical communities worldwide. Fortunately, with the pandemic mainly behind us, we hope to fulfil the initial aim of the conference of becoming a major bi-annual event of the region's mathematical calendar, answering the regional need for deeper ties and more intensive scientific exchange.

The organizers of the second Joint mathematical meeting of Serbia and Montenegro are

Mathematical Institute of the Serbian Academy of Sciences and Arts
Faculty of Mathematics in Belgrade
Faculty of Natural Sciences and Mathematics in Podgorica
Matica srpska - Association of Members in Montenegro
Mathematical Forum of Montenegro.

The conference is supported by Ministry of Science, Technological Development and Innovation of Republic of Serbia, Raiffeisen Bank, Center for Investments and Finance (CIF) and Foundation Nemanja Trifunović.

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Abstracts

Missing data: the impact on multivariate goodness-of-fit tests

Danijel Aleksić

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Bojana Milošević

University of Belgrade, Faculty of Mathematics

Missing data is one of the most commonly met problems in data analysis. Therefore it has attracted the attention of researchers from different scientific fields. The main focus of the research has been on developing adequate imputation procedures whose quality has been assessed in the context of predictive properties of different models of interest. However, the impact of missingness on the data distribution has not been studied thoroughly in the context of testing goodness-of-fit with multivariate distributions. In this talk, we present some of the recent developments in this direction and some new open questions regarding the topic.

Locally strongly convex affine hypersurfaces that admit generalized Calabi decomposition

Miroslava Antić

University of Belgrade, Faculty of Mathematics

In affine differential geometry, there is a well known construction, discovered by Calabi, of obtaining a new affine hyperbolic hypersphere starting with two hyperbolic affine hyperspheres or a hypersphere and a point. Intrinsically, the new hypersphere is the product of two original hyperspheres and a one-dimensional factor with image which is a special planar curve. This construction can be generalized by taking arbitrary planar curve. Such hypersurfaces admit a special decomposition of their tangent bundle into two or three distributions, defined in terms of the shape operator and the difference tensor and have a particular warped product structure. We will present the characterisation of the locally strongly convex affine hypersurfaces that can be decomposed as a generalized Calabi product of two affine hyperspheres or a hypersphere and a point. Further, we will study hypersurfaces with the constant sectional curvature whose shape operator has at most one eigenvalue of multiplicity one, show that they admit the generalized Calabi decomposition and give their classification.

O reprezentaciji kompleksnog polinoma kao sume monoma

Radoš Bakić

Univerzitet u Beogradu, Fakultet za obrazovanje učitelja i vaspitača

Diskutuje se problem reprezentacije kompleksnog polinoma kao suma monoma, i daje se ocena parametara koji se pojavljuju. Takođe će biti dat kratak prikaz istorije ove oblasti (geometrija polinoma), uključujući neka aktuelna pitanja.

A novel class of independence tests designed for bivariate count data

Dana Bucalo Jelić

University of Belgrade, Faculty of Agriculture

A novel class of independence tests based on probability generating functions (p.g.f.) is proposed for bivariate count data. The empirical counterpart of the difference between joint and the product of marginal p.g.f. can be represented as a power series. Using the weighted l^2 norm to the coefficients of the power series leads to the novel class of independence test. The wide empirical study shows that the novel (proposal) class keeps the competitiveness with respect to some existing ones.

Finding all the extremal trees corresponding to a given degree sequence for certain functions on adjacent vertex degrees

Ivan Damnjanović

University of Niš, Faculty of Electronic Engineering

Diffine LLC

Wang [*Cent. Eur. J. Math.* 12:1656–1663, 2014] has considered any discrete symmetric function $\mathcal{F} : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{R}$ such that

$$\mathcal{F}(x, a) + \mathcal{F}(y, b) \geq \mathcal{F}(y, a) + \mathcal{F}(x, b) \quad \text{for any } x \geq y \text{ and } a \geq b,$$

and showed that, among all the trees \mathcal{T}_D corresponding to a given degree sequence D , the greedy tree must attain the maximum value of the graph invariant given by the expression

$$\sum_{u \sim v} \mathcal{F}(\deg(u), \deg(v)),$$

where $\deg(u)$ and $\deg(v)$ represent the degrees of the vertices u and v , respectively, and the summing is performed across all the unordered pairs of adjacent vertices u and v . In this presentation, we extend Wang's result by determining all the possible trees from \mathcal{T}_D that attain the maximum value of the aforementioned graph invariant, thereby fully resolving the corresponding extremal problem.

Neighborhood management in variable intensified neighborhood search

Tatjana Davidović
Mathematical Institute SASA

Spomenka Milić
University of Belgrade, Faculty of Mathematics

Variable Intensified Neighborhood Search (VINS) matheuristic is an optimization method proposed in 2015. VINS is a hybrid heuristic that combines a Mixed Integer Programming (MIP) exact solver with neighborhood-based heuristic search procedure. The main idea behind VINS is fixing a subset of variables using heuristic principles and then invoking a time-limited exact solver to determine values for the remaining variables. To change the number and structure of variables that should be fixed, different types and sizes of neighborhoods are explored. As the search advances, the size of neighborhood is systematically increased (together with the corresponding time limit for MIP solver). This yields the increase in the intensity of the search and that is the inspiration for the method's name. VINS explores 10 types of neighborhoods in the pre-specified order. Although shown very efficient, with respect to other matheuristics, on numerous optimization problems, sometimes it may be too slow to explore all the neighborhoods. Therefore, our goal is to examine how each particular neighborhood influences on the quality of final solution, to identify the robust subset of best-performing neighborhoods, and to find the appropriate order of neighborhoods from the identified subset. By the term "robust" we mean the subset of neighborhood that performs the best on the majority of tested examples.

Iterated integrals of modular forms and multiple L -values

Goran Djanković
University of Belgrade, Faculty of Mathematics

We will describe certain multiple L -functions associated to several holomorphic modular forms. We show analytic continuation and the functional equation of these series through iterated period integrals. Moreover we discuss connections with multiple Eisenstein series, Eichler integrals and quantum modular forms.

Some new approaches to summability factors

Ivana Djolović
University of Belgrade, Technical faculty in Bor

Here we present recent research related to known results on summability factors and general summability methods but this time applying the theory of matrix transformations and classical sequence spaces.

Dvostruko stohastička i permutaciona rešenja jednačine $AXA = XAX$

Bogdan Đorđević

Matematički institut Srpske akademije nauka i umetnosti

Za datu permutacionu matricu A , okarakterisaćemo potrebne i dovoljne uslove za egzistenciju dvostruko stohastičkih i permutacionih rešenja jednačine $AXA = XAX$. Prikazaćemo algoritam za pronalaženje svih permutacionih rešenja, kao i algoritam za nalaženje nekih od dvostruko stohastičkih rešenja. Rezultati će biti praćeni i numeričkim primerima.

Homomorphisms on Banach modules

Dragan S. Djordjević

University of Niš, Faculty of Sciences and Mathematics

We present results related to homomorphisms on Banach modules which are related to the Hahn-Banach extension theorem, dual spaces and Fredholm theory. Thus, some well-known results are extended to a more general setting.

Yamabe solitons on CR submanifolds of maximal CR dimension in Kähler manifolds

Miloš Djorić, Mirjana Djorić

University of Belgrade, Faculty of Mathematics

R. Hamilton introduced the notions of Ricci and Yamabe flows, in which the metric on a Riemannian manifold is deformed by evolving according to flow equations. Under suitable conditions, the Ricci flow evolves an initial metric to an Einstein metric while the Yamabe flow evolves an initial metric to a new one with constant scalar curvature within the same conformal class. Ricci and Yamabe solitons correspond to self-similar solutions of the Ricci and Yamabe flow.

In the joint work with Mirjana Djorić, we give the necessary and sufficient conditions for a CR submanifold M of maximal CR dimension in arbitrary Kähler manifold $(\tilde{M}, \tilde{g}, J)$, with unit normal vector field ξ , to admit Yamabe soliton with Reeb vector field $U = -J\xi$ as a soliton vector field. Furthermore, we obtain the complete classification of such solitons M when the ambient manifold \tilde{M} is a non-flat complex space form.

Topologically sensitive Variable Neighborhood Search - Design and the initial results

Vladimir Filipović, Aleksandar Kartelj
University of Belgrade Faculty of Mathematics

Design of the novel metaheuristic algorithm, named Topologically sensitive Variable Neighborhood Search, is presented, and its implementation is elaborated. The proposed metaheuristic algorithm is applied to well-known NP-hard problem: Minimum Set Cover Problem. Relevant computational experiments are prepared and executed, and obtained results are presented and discussed.

Type system for trustworthy formal models

Silvia Gilezan
University of Novi Sad and Mathematical Institute SASA

The Curry-Howard correspondence connects logic and computation wherein mathematical proofs coincide with computer programs (terms) and formulae with types. With this respect, types have gained an important role in the analysis of formal systems. A type system splits elements (terms, programs) of a language, into sets, called types, and proves absence of certain undesired behaviors. In programming languages, types represent a well-established technique to ensure program correctness. Accordingly, types have provided trustworthiness to foundations of programming languages. Nowadays, there is a plethora of type systems in logic, programming languages, distributed, concurrent and large scale systems which establish trustworthiness of the particular framework. In this talk, we present some significant type systems - functional types, behavioural types, session types, linked data types - along with their properties such as type safety, liveness, deadlock freedom and discuss their role for ensuring trustworthiness of corresponding formal models.

Mathematical distant relations and the EPR paradox

Ivana D. Ilić, Jelena M. Višnjić
University of Niš, Faculty of Medicine

Branislav Randjelović
University of Niš, Faculty of Electronic Engineering

Vojislav V. Mitić
University of Niš, Faculty of Electronic Engineering
Institute of Technical Sciences SASA

Luka Ćirić
EPFL Lausanne, Department for the Experimental Physics

On the given topological space (X, τ) we define a notion of τ -metric, such that the set X is a τ -metric space and we prove some properties of these spaces. We construct mathematical model of the distant space correlations between quantum systems that were initially allowed to interact and then separated. The main idea of the paper arises from the EPR paradox, the paradox of Einstein, Podolsky and Rosen that occurs when measurement of a physical observable performed on one system have an immediate effect on the other separate system being entangled with it. The proposed mathematical model of these relations may be attractive for future researches in the field of microphysics.

Hiperciklični operatori na prostoru mera

Stefan Ivković

Matematički institut Srpske akademije nauka i umetnosti

Dinamika težinskih kompozicionih operatora na različitim prostorima funkcija je bila proučavana u brojnim radovima. Ukoliko ovi operatori deluju na prostor $C_0(X)$ gde je X lokalno kompaktni Hausdorfov topološki prostor, onda konjugovani operatori ovih operatora deluju na Banahovom prostoru regularnih Borelovih mera na X . U ovom radu dajemo neophodne i dovoljne uslove da konjugovani operatori težinskih kompozicionih operatora budu hiperciklični na prostoru regularnih Borelovih mera na X . Osim toga dajemo dovoljne uslove da ovi operatori budu haotični i rezultate ilustrujemo konkretnim primerima.

Optimizacija. Varijacione nejednakosti.

Milojica Jaćimović

Crnogorska akademija nauka i umjetnosti

Nevena Mijajlović

Univerzitet Crne Gore, Prirodno-matematički fakultet

U ovom predavanju biće predstavljeni osnovni zadaci optimizacije i varijacione nejednakosti, uslovi egzistencije, neophodni uslovi optimuma i numeričke metode gradijentnog tipa.

Collecting two sets of coupons: some real-life interpretations

Jelena Jocković, Bojana Todić
University of Belgrade, Faculty of Mathematics

The classical coupon collector's problem has various generalizations, and some of them can be easily explained through real-life examples. One of these generalizations is dealing with the case of collecting two sets of coupons (by two independent collectors, say, brother and sister). The existing results are related to the scenario when sister is the main collector, and when she gets a "double", she gives it to her brother to complete his own collection. The case of collecting several collections, by independent collectors, is also well known. We will use Markov chain approach to define and analyze the following scenarios: 1. brother and sister are buying coupons independently, but they agree to exchange "doubles"; 2. when one of them completes the collection, they both keep buying coupons in order to complete the other collection. The object of interest is the mean waiting time until one or both collections are completed. Some of the results will be generalized to the case of collecting several collections.

On the uniqueness of solutions to stochastic scalar conservation laws on manifolds

Nikola Konatar
University of Montenegro, Faculty of Science and Mathematics

We consider stochastic scalar conservation laws on compact manifolds, giving admissibility conditions and the kinetic formulation of the problem. We use these to give a new proof for the uniqueness of solutions of these equations.

BE-algebras of small order

Nina Konjikušić, Miloš Mićović, Ivana Jovović
University of Belgrade, School of Electrical Engineering

In this paper, we consider BE-algebras with some of their properties. The examples of different types (self-distributive, implicative, transitive, and commutative) of BE-algebras are formed using the programming languages Python and Julia. The number of non-isomorphic BE-algebras of the order less than 6 is obtained.

Enumeration of maximum matchings in the Tower of Hanoi graphs and truncated Hanoi graphs

Žana Kovijanić Vukićević

University of Montenegro, Faculty of Science and Mathematics

We consider Tower of Hanoi graphs and study their matching properties. Explicit system of recurrences are derived for the matching polynomials of these graphs and their appropriate truncated variants. Consequently, we obtain exact formula for the numbers of maximum matchings in Hanoi graphs using matching polynomials, that is a new approach for old one problem.

Lipschitz conditions and real harmonic functions on convex polyhedra

Marijan Marković

University of Montenegro, Faculty of Science and Mathematics

We consider Lipschitz conditions on a real-valued harmonic function on a convex polyhedron in \mathbb{R}^n . Let $\Pi \subseteq \mathbb{R}^n$ be a such domain. Let U be a real harmonic function on Π . Our main result says that if there exists a constant C such that

$$||U|(\zeta) - |U|(\zeta - r\mathbf{n})| \leq Cr^\alpha, \quad \zeta \in \partial\Pi, r > 0, \zeta - r\mathbf{n} \in \Pi,$$

where \mathbf{n} is among all outward normals on the boundary of the polyhedron, then there exists a constant \tilde{C} such that

$$|U(x) - U(y)| \leq \tilde{C}|x - y|^\alpha, \quad x, y \in \bar{\Pi}.$$

Here $\alpha \in (0, 1)$ is arbitrary.

On the empirical probability generating function based goodness-of-fit tests for count data

Bojana Milošević

University of Belgrade, Faculty of Mathematics

A convenient way to characterize count data distributions is by their probability-generating functions. Therefore, its empirical counterpart, the so-called empirical probability generating function, takes an important place in the construction of goodness-of-fit (GOF) tests. During the talk, we focus on several types of characterizations for discrete data and for each, show general approaches for the construction of GOF test statistics, accompanied by their large sample properties. The methodology is illustrated with several recent examples of GOF tests. A part of the talk is dedicated to the overview of some working results and some potential directions for future research.

Stochastic serotonin model: the existence, uniqueness and positivity of the global solution

Marija Milošević

University of Niš, Faculty of Sciences and Mathematics

Serotonin, as a hormone and neurotransmitter, is significant in many areas of science, such as biology, neurology, psychiatry, among others. The subject of this talk is a mathematical model of serotonergic system including generation of serotonin, action-potential release of serotonin into the synapse, reuptake of serotonin and its metabolic breakdown. The model incorporates deterministic as well as random influences on the serotonin intracellular and extracellular dynamics, such that it is based on a nonautonomous system of stochastic differential equations with discontinuous drift coefficient. The main results which will be presented are the existence, uniqueness and positivity of the global solution of that system.

Product of chain connected sets in a space

Zoran Misajleski, Emin Durmishi, Aneta Velkoska
Ss. Cyril and Methodius University in Skopje

In this paper we provide a new results for the products of chain connected sets in a topological space [1, 2] and we study the properties of the products of the chain connected sets in a space that is more general then a topological space and consists of a set and family of coverings of the set.

Gradient neural dynamics for solving system of matrix equations and their applications

Marko Petković

University of Niš, Faculty of Sciences and Mathematics

Jelena Dakić

University of Montenegro, Faculty of Science and Mathematics

We consider the gradient neural network models for solving several different systems of linear matrix equations in areal time. The convergence properties are investigated in details using Lyapunov method and it is shown that models are globally convergent to the general solution, determined by the initial values choice. A few applications are given, including the computation of matrix generalized inverses. Several numerical examples are shown to illustrate the theoretical results.

Kratki prikaz prevođenja Hegorovih dijagrama u hirurgiju

Zoran Petrić, Mladen Zekić
Matematički institut Srpske akademije nauka i umetnosti

Jovana Nikolić
Univerzitet u Beogradu, Matematički fakultet

Predstavićemo najvažnije korake koji od Hegorovog cepanja mnogostrukosti dovode do njenog zadavanja pomoću hirurgije.

On arithmetic–geometric index of chemical trees and unicyclic graphs

Goran Popivoda, Žana Kovijanić Vukićević
University of Montenegro, Faculty of Science and Mathematics

Saša Vujošević
University of Montenegro, Faculty of Economics

In this report, we characterize chemical trees and unicyclic graphs extremal concerning the arithmetic-geometric index:

$$AG(G) = \sum_{uv \in E(G)} \frac{d(u) + d(v)}{2\sqrt{d(u)d(v)}}.$$

Also, we present the relations between the $AG(G)$ and its predecessor geometric–arithmetic index $GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d(u)d(v)}}{d(u)+d(v)}$.

Pseudo linear combination of fuzzy metrics

Nebojša M. Ralević
University of Novi Sad, Faculty of Technical Sciences

Bratislav Iričanin
University of Belgrade, Faculty of Electrical Engineering

Dejan Čebić
University of Belgrade, Faculty of Mining and Geology

A new fuzzy metric from already defined fuzzy metrics defined over the same set using by a pseudo linear combination is constructed in this paper. Operations used in pseudo linear combination are triangular norm and conorm. The fuzzy space thus obtained is proved to be complete. Additional features related to this space are also shown. The fuzzy metric obtained in this way, from the fuzzy metrics used for the spatial distance and the color similarity measured between pixels in the image, can be used to construct an image denoising procedure. The goal was to improve the sharpness and the quality of the image, which are expressed and measured by means of the image quality index.

Two relations on the set of (b, c) -invertible elements in rings

Ivana Stanišev

University of Belgrade, Technical Faculty in Bor

Jelena Višnjić

University of Niš, Faculty of Medicine

Dragan S. Djordjević

University of Niš, Faculty of Sciences and Mathematics

In recent years there is a number of papers that study partial orders based on generalized inverses in rings. Inspired by minus, sharp, core and dual core partial orders, we introduce two binary relations induced by (b, c) -invertible elements in rings. It turns out that these are the equivalence relations. Beside, we give some properties of elements that are in these relations..

Statistical analysis of fitting Lognormal distribution with Benford's Law

Jelena Stanojević, Dragana Radojčić, Vesna Rajić, Tatjana Rakonjac-Antić

University of Belgrade, Faculty of Economics and Business

As Benford's Law is well known in the literature as a method for detecting some irregularity in the data set, we investigate different properties of that law, to confirm its application. We use the fact that Lognormal distribution under some parameter constraints obeys Benford's Law and we confirm that on the real data set (in which the Lognormal distribution fits well). We also confirm that with some simulation studies.

Reinforcement learning for graphs

Dragan Stevanović

Mathematical Institute SASA

Salem Al-Yakoob, Mohammad Ghebleh, Ali Kanso

Kuwait University, Faculty of Science, Department of Mathematics

Adam Wagner [arXiv:2104.14516 (2021)] recently suggested a way of using reinforcement learning for constructing (counter)examples in graph theory. Here we will present a more user-friendly reimplementation of the Wagner's approach and illustrate its use on a few graph-theoretical problems.

Simetrična funkcija digrafa

Tanja Stojadinović, Vladimir Grujić
University of Belgrade, Faculty of Mathematics

U nizu predavanja koje je održao prošlog leta, Ričard Stenli je uveo simetričnu funkciju pridruženu digrafu. Ona broji skup padova permutacija koji odgovaraju datom digrafu. U ovom izlaganju pokazaćemo da je ova simetrična funkcija zapravo indukovana izvesnom strukturom kombinatorne Hopfove algebre na digrafovima.

Coupon collection problem: mean waiting time to complete collection

Bojana Todić
University of Belgrade, Faculty of Mathematics

The combinatorial problem of collecting coupons has been considered since the 60s and it is still an active field of research. In this paper, several different generalizations of this problem will be presented, and the mean waiting time required to complete a collection of the coupons will be analyzed. Also, numerical comparisons of the coupon collection speed for different variants of the coupon collection problem will be presented.

The application of BlockChain technology in Maritime Supply Chain Systems

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Slobodan Jelić
University of Belgrade, Faculty of Civil Engineering
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Miodrag Mihaljević
Mathematical Institute SASA

BlockChain (BC) represents decentralized, distributed, append-only data storage. Decentralization refers to the removal of centralized authority that manages the storage. This role take the users of the BC who are managing it by executing the consensus protocol. Distribution of BC means that each user of the BC possesses a copy of the database, in order to ensure the wider availability of the data stored in the BC. Append-only means that the data stored in the BC cannot be modified or deleted, only new data can be added. Because of these features, the main applicability of BC was in the field of cryptocurrencies. However, with the addition of smart contracts, the BC transformed in a general-purpose decentralized platform.

Since then, BC has been applied in various areas. One of such applications is the usage of BC for supply chain management, providing a decentralized way to track and trace the transported goods from their origins to the destinations. Although the application of BC will increase over time, its main drawback is the huge amount of resources needed to execute the consensus protocol. This makes the BC management expensive and energy consumable. We propose a platform that uses the BC technology to support the maritime supply chain systems (MSCSs) in order to mitigate the existing issues of those system such as product traceability, complex documentation and trust issues between parties involved. In addition, we propose the usage of the novel Combinatorial Optimization Consensus Protocol (COCP) that would allow solving instances of combinatorial optimization problems. COCP allows solving instances of the real-life problems that originate from MSCS, while simultaneously managing the BC based platform. The presented work is a result of the Montenegro-Serbia Project "Development of innovative AI based solution for a Blockchain technology in Maritime Supply chain" (ABMarSupply) from the SMART4ALL Horizon 2020 project: Bilateral Knowledge Transfer Experiment, and the "Advanced Artificial Intelligence Techniques for Analysis and Design of System Components Based on Trustworthy BlockChain Technology" (AI4TrustBC) project supported by the Science Fund of the Republic of Serbia.

On neutral stochastic functional differential equations with highly nonlinear coefficients

Teodora Trifunović

University of Kragujevac, Faculty of Science

Miljana Jovanović, Marija Milošević

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Neutral stochastic functional differential equations (NSFDEs), as a generalization of stochastic functional differential equations, are recognized and studied by many authors under the various conditions when it comes to proving the existence and uniqueness of the solution. As many NSFDEs have highly nonlinear coefficients, which do not satisfy the global Lipschitz conditions, in general case, whole consideration is based on the Khasminskii-type conditions together with local non-Lipschitz condition. Different forms of initial condition are considered: one defined on finite and other defined on infinite interval. Moment estimates and generalized Lyapunov exponent are obtained.

Some properties of (b, c) -inverses in rings

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In this talk, we consider when some well known properties of generalized inverses and the classical inverse, hold for (b, c) -inverses, in a ring with the unity 1. Also, we prove that in the case when the unity is (b, c) -invertible, the set of all (b, c) -inverses is a group, with respect to multiplication.

Kurt Gödel – From mathematics to philosophy

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Among of the most interesting philosophical mysteries of our time are incompleteness theorems. They states that there are limits of provability in formal mathematical theories - in any reasonable mathematical system there are always true statements that cannot be proved. The meaning of those theorems were examined by many philosophers and logicians, but always with reserve and criticism to the interpretation given by their author, Kurt Gödel (1906 - 1978). He understood them as a strong argument in support of the existence of the conceptual world. Although quite authentic, such an interpretation was by no means acceptable to philosophers of the twentieth century. Gödel's interpretation was looked down upon, with materialistic and positivist prejudices, as old fashioned, and outdated metaphysical position. The interest in Gödel's results was enormous, they raised logic to a completely new level, but Gödel remained quite alone in his interpretation of them. As he didn't find any help from the philosophers he took upon himself to illuminate "one of the most important philosophical problem - the problem of existence of the conceptual world." As he did not find understanding among his contemporaries, he worked all alone, seeking justification for his ideas and viewpoints from Leibniz, Kant and finally from Husserl. Over time, he became lonely and isolated and was not willing to publish his research for more then thirty years and gave his last lecture in 1951. Among philosophers with whom he discussed his views was Rudolph Carnap, the most important philosopher of the Vienna Circle, to which Gödel himself belonged at the beginning of his career. Some interesting details of their relatively long lasting contacts will be presented in this lecture.

Topological methods in combinatorics and discrete geometry through the eyes of CGTA seminar

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The method of configuration spaces and test maps, better known as the “Configuration space/test map scheme” (CS/TM), has been for decades one of central methods for applying Algebraic Topological methods in Combinatorics and discrete Geometry.

<http://www.csun.edu/~ctoth/Handbook/chap21.pdf>

<https://www.msri.org/programs/309>

<https://mccme.ru/dubna/2018/courses/zivaljevic.html>

(CS/TM) has been to the present day one of the central topics and a guiding principle of Belgrade CGTA seminar, which was founded more than 35 years ago by a group of Belgrade math enthusiasts. The lecture will try to recreate the atmosphere of the early days of CGTA seminar and cover some more recent developments and new applications of the method.
